第3章 3 「連続型確率分布」「連続型確率変数の平均と分散」 第2回

解答

1. $a = 1$, $P(0.5 \leqq X \leqq 1.5) = 0.75 \left(= \dfrac{3}{4}\right)$

2. $P(-1 \leqq X \leqq 1) = \dfrac{1}{4}$, $E[X] = 2$, $V[X] = \dfrac{4}{3}$

解説

1. $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_0^2 f(x)\,dx$

$$= \int_0^1 ax\,dx + \int_1^2 a(2-x)\,dx$$

$$= a\left[\frac{x^2}{2}\right]_0^1 + a\left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \frac{a}{2} + \frac{a}{2} = a = 1 \text{（全確率）} \quad \therefore \quad a = 1$$

$$P(0.5 \leqq X \leqq 1.5) = \int_{0.5}^{1.5} f(x)\,dx$$

$$= \int_{0.5}^{1} x\,dx + \int_1^{1.5} (2-x)\,dx$$

$$= \frac{(0.5+1)\times 0.5}{2} \times 2 = 0.75$$

2. $\displaystyle P(-1 \leqq X \leqq 1) = \int_{-1}^{1} f(x)\,dx = \int_0^1 \frac{1}{4}\,dx = \frac{1}{4}$

$$E[X] = \int_{-\infty}^{\infty} xf(x)\,dx$$

$$= \int_0^4 x\frac{1}{4}\,dx$$

$$= \frac{1}{4}\left[\frac{x^2}{2}\right]_0^4 = \frac{8}{4} = 2$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_0^4 x^2\frac{1}{4}\,dx$$

$$= \frac{1}{4}\left[\frac{x^3}{3}\right]_0^4 = \frac{1}{4} \times \frac{64}{3} = \frac{16}{3}$$

$$V[X] = E[X^2] - \big(E[X]\big)^2 = \frac{16}{3} - 2^2 = \frac{4}{3}$$