第 3 章 3 「連続型確率分布」「連続型確率変数の平均と分散」 第 1 回

解答

1. $a = 1$，$P(0.5 \leqq X \leqq 1.5) = \dfrac{1}{8}$

2. $P(0 \leqq X \leqq 2) = \dfrac{4}{27}$，$E[X] = 0$，$V[X] = \dfrac{27}{5}$

解説

1. $\displaystyle\int_{-\infty}^{\infty} f(x)\,dx = \int_{-1}^{1} f(x)\,dx$

$\displaystyle = \int_{-1}^{0} a(1+x)\,dx + \int_{0}^{1} a(1-x)\,dx$

$\displaystyle = a\left[x + \frac{x^2}{2}\right]_{-1}^{0} + a\left[x - \frac{x^2}{2}\right]_{0}^{1}$

$\displaystyle = \frac{a}{2} + \frac{a}{2} = a = 1$(全確率) $\quad \therefore \quad a = 1$

$\displaystyle P(0.5 \leqq X \leqq 1.5) = \int_{\frac{1}{2}}^{1} f(x)\,dx$

$\displaystyle = \int_{\frac{1}{2}}^{1} 1 \cdot (1-x)\,dx = \left[x - \frac{x^2}{2}\right]_{\frac{1}{2}}^{1}$

$\displaystyle = \left(1 - \frac{1}{2}\right) - \left(\frac{1}{2} - \frac{1}{8}\right) = \frac{1}{8}$

2. $\displaystyle P(0 \leqq X \leqq 2) = \int_{0}^{2} \frac{1}{18}x^2\,dx$

$\displaystyle = \frac{1}{18}\left[\frac{x^3}{3}\right]_{0}^{2} = \frac{4}{27}$

$\displaystyle E[X] = \int_{-\infty}^{\infty} xf(x)\,dx$

$\displaystyle = \int_{-3}^{3} x\frac{1}{18}x^2\,dx$

$= 0$ ($\because$ 奇関数の積分の性質)

$\displaystyle E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$\displaystyle = \int_{-3}^{3} x^2\frac{1}{18}x^2\,dx$

$\displaystyle = 2\int_{0}^{3} \frac{1}{18}x^4\,dx$ ($\because$ 偶関数の積分の性質)

$\displaystyle = \frac{2}{18}\left[\frac{x^5}{5}\right]_{0}^{3} = \frac{27}{5}$

$\displaystyle V[X] = E[X^2] - \big(E[X]\big)^2 = \frac{27}{5} - 0^2 = \frac{27}{5}$