

第3章 1 「確率変数と確率分布」 第3回

解答

1. (1) $\frac{7}{2}$

(2) $\frac{35}{12}$

(3) $\frac{35}{12}$

(4) $\frac{\sqrt{105}}{6}$

2. (1) i) $\frac{1}{4}$, ii) $\frac{1}{2}$, iii) $\frac{1}{4}$

(2) $\frac{9}{2}$

(3) $\frac{19}{4}$

(4) $\frac{\sqrt{19}}{2}$

3. $E\left[\frac{X-1}{2}\right] = 1$, $V\left[\frac{X-1}{2}\right] = 1$

4. (1) $\mu a + b$

(2) $\sigma^2 a^2$

(3) $a = \frac{1}{\sigma}$, $b = -\frac{\mu}{\sigma}$

解説

1. (1) $E[X] = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = \frac{7}{2}$

(2) $V[X] = \left(1 - \frac{7}{2}\right)^2 \times \frac{1}{6} + \left(2 - \frac{7}{2}\right)^2 \times \frac{1}{6} + \left(3 - \frac{7}{2}\right)^2 \times \frac{1}{6} + \left(4 - \frac{7}{2}\right)^2 \times \frac{1}{6} + \left(5 - \frac{7}{2}\right)^2 \times \frac{1}{6} + \left(6 - \frac{7}{2}\right)^2 \times \frac{1}{6} = \frac{35}{12}$

(3) $E[X^2] = 1^2 \times \frac{1}{6} + 2^2 \times \frac{1}{6} + 3^2 \times \frac{1}{6} + 4^2 \times \frac{1}{6} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{1}{6} = \frac{91}{6}$

$$V[X] = E[X^2] - (E[X])^2 = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$$

(4) $\sigma = \sqrt{\frac{35}{12}} = \frac{\sqrt{105}}{6}$

2. (1) i) $X = 2$ となるのは、くじの番号が1

の場合であるから $P(X = 2) = \frac{1}{4}$

ii) $X = 4$ となるのは、くじの番号が2または3

の場合であるから $P(X = 4) = \frac{2}{4} = \frac{1}{2}$

iii) $X = 8$ となるのは、くじの番号が4

の場合であるから $P(X = 8) = \frac{1}{4}$

(2) $E[X] = 2 \times \frac{1}{4} + 4 \times \frac{2}{4} + 8 \times \frac{1}{4} = \frac{9}{2}$

(3) $E[X^2] = 2^2 \times \frac{1}{4} + 4^2 \times \frac{2}{4} + 8^2 \times \frac{1}{4} = 25$

$$V[X] = E[X^2] - (E[X])^2 = 25 - \left(\frac{9}{2}\right)^2 = \frac{19}{4}$$

(4) $\sigma = \sqrt{\frac{19}{4}} = \frac{\sqrt{19}}{2}$

3. $E[3X + 1] = 3E[X] + 1 = 10$ より $E[X] = 3$

$$V[3X + 1] = 9V[X] = 36$$
 より $V[X] = 4$

$$E\left[\frac{X-1}{2}\right] = \frac{1}{2}E[X] - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = 1$$

$$V\left[\frac{X-1}{2}\right] = \left(\frac{1}{2}\right)^2 V[X] = \frac{1}{4} \times 4 = 1$$

4. (1) $E[Y] = E[aX + b] = aE[X] + b = \mu a + b$

(2) $V[Y] = V[aX + b] = a^2 V[X] = \sigma^2 a^2$

(3) $\sigma^2 a^2 = 1$, $a > 0$, $\sigma > 0$ より $a = \frac{1}{\sigma}$

$$\mu a + b = 0$$
 に $a = \frac{1}{\sigma}$ を代入して $b = -\frac{\mu}{\sigma}$