

第3章 1 「確率変数と確率分布」 第2回

解答

1. (1) $\frac{3}{2}$

(2) $\frac{3}{4}$

(3) $\frac{3}{4}$

(4) $\frac{\sqrt{3}}{2}$

2. (1) i) $\frac{1}{3}$, ii) $\frac{1}{6}$

(2) $\frac{3}{2}$

(3) $\frac{11}{12}$

(4) $\frac{\sqrt{33}}{6}$

3. $E\left[\frac{Y}{3} - 8\right] = 0$, $V\left[\frac{Y}{3} - 8\right] = 1$

4. (1) $24a + b$

(2) $9a^2$

(3) $a = 2, b = 48$

4. (1) $E[Y] = E[aX + b] = aE[X] + b = 24a + b$

(2) $V[Y] = V[aX + b] = a^2V[X] = 9a^2$

(3) $9a^2 = 36, a > 0$ より $a = 2$

$24a + b = 96$ に $a = 2$ を代入して $b = 48$

解説

1. (1) $E[X] = 0 \times \frac{1}{8} + 1 \times \frac{3}{8} + 2 \times \frac{3}{8} + 3 \times \frac{1}{8} = \frac{3}{2}$

(2) $V[X] = \left(0 - \frac{3}{2}\right)^2 \times \frac{1}{8} + \left(1 - \frac{3}{2}\right)^2 \times \frac{3}{8}$
 $+ \left(2 - \frac{3}{2}\right)^2 \times \frac{3}{8} + \left(3 - \frac{3}{2}\right)^2 \times \frac{1}{8} = \frac{3}{4}$

(3) $E[X^2] = 0^2 \times \frac{1}{8} + 1^2 \times \frac{3}{8} + 2^2 \times \frac{3}{8} + 3^2 \times \frac{1}{8} = 3$

$V[X] = E[X^2] - (E[X])^2 = 3 - \left(\frac{3}{2}\right)^2 = \frac{3}{4}$

(4) $\sigma = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

2. (1) i) $X = 1$ となるのは、さいころの目が 2, 3 の

場合であるから $P(X = 1) = \frac{2}{6} = \frac{1}{3}$

ii) $X = 3$ となるのは、さいころの目が 6 の

場合であるから $P(X = 3) = \frac{1}{6}$

(2) $E[X] = 0 \times \frac{1}{6} + 1 \times \frac{2}{6} + 2 \times \frac{2}{6} + 3 \times \frac{1}{6}$

$= \frac{9}{6} = \frac{3}{2}$

(3) $E[X^2] = 0^2 \times \frac{1}{6} + 1^2 \times \frac{2}{6} + 2^2 \times \frac{2}{6} + 3^2 \times \frac{1}{6}$

$= \frac{19}{6}$

$V[X] = E[X^2] - (E[X])^2$

$= \frac{19}{6} - \left(\frac{3}{2}\right)^2 = \frac{11}{12}$

(4) $\sigma = \sqrt{\frac{11}{12}} = \frac{\sqrt{33}}{6}$

3. $E\left[\frac{X}{3} - 8\right] = \frac{1}{3}E[X] - 8 = \frac{1}{3} \times 24 - 8 = 0$

$V\left[\frac{X}{3} - 8\right] = \left(\frac{1}{3}\right)^2 V[X] = \frac{1}{9} \times 9 = 1$