

第3章 1 「確率変数と確率分布」 第1回

解答

1. (1) 0 (2) $\frac{7}{4}$

(3) $\frac{7}{4}$ (4) $\frac{\sqrt{7}}{2}$

2. (1) i) $\frac{3}{10}$, ii) $\frac{3}{5}$ (2) $\frac{4}{5}$

(3) $\frac{9}{25}$ (4) $\frac{3}{5}$

3. $E[3X - 1] = 29$, $V[3X - 1] = 18$

4. (1) $8a + b$ (2) $\frac{16}{9}a^2$

(3) $a = \frac{3}{4}$, $b = -6$

解説

1. (1) $E[X] = (-2) \times \frac{1}{8} + (-1) \times \frac{3}{8} + 1 \times \frac{3}{8} + 2 \times \frac{1}{8}$
 $= 0$

(2) $V[X] = (-2 - 0)^2 \times \frac{1}{8} + (-1 - 0)^2 \times \frac{3}{8}$
 $+ (1 - 0)^2 \times \frac{3}{8} + (2 - 0)^2 \times \frac{1}{8}$
 $= \frac{14}{8} = \frac{7}{4}$

(3) $E[X^2] = (-2)^2 \times \frac{1}{8} + (-1)^2 \times \frac{3}{8} + 1^2 \times \frac{3}{8}$
 $+ 2^2 \times \frac{1}{8} = \frac{14}{8} = \frac{7}{4}$

$V[X] = E[X^2] - (E[X])^2 = \frac{7}{4} - 0^2 = \frac{7}{4}$

(4) $\sigma = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{2}$

2. (1) i) $P(X = 0) = \frac{{}_2C_0 \times {}_3C_2}{{}_5C_2} = \frac{3}{10}$

ii) $P(X = 1) = \frac{{}_2C_1 \times {}_3C_1}{{}_5C_2} = \frac{6}{10} = \frac{3}{5}$

(2) $E[X] = 0 \times \frac{3}{10} + 1 \times \frac{6}{10} + 2 \times \frac{1}{10} = \frac{4}{5}$

(3) $E[X^2] = 0^2 \times \frac{3}{10} + 1^2 \times \frac{6}{10} + 2^2 \times \frac{1}{10} = 1$

$V[X] = E[X^2] - (E[X])^2 = 1 - \left(\frac{4}{5}\right)^2 = \frac{9}{25}$

(4) $\sigma = \sqrt{\frac{9}{25}} = \frac{3}{5}$

3. $E[3X - 1] = 3E[X] - 1 = 3 \times 10 - 1 = 29$

$V[3X - 1] = 3^2 V[X] = 9 \times 2 = 18$

4. (1) $E[Y] = E[aX + b] = aE[X] + b = 8a + b$

(2) $V[Y] = V[aX + b] = a^2 V[X] = \frac{16}{9}a^2$

(3) $\frac{16}{9}a^2 = 1$, $a > 0$ より $a = \frac{3}{4}$

$8a + b = 0$ に $a = \frac{3}{4}$ を代入して $b = -6$